DPP

DPP No. 01 [Solution]
Topic : Permutation & Combinations
Year : 2023

1. The total number of selections of fruit which can be made from 3 bananas, 4 apples and 2 oranges, is
a) 39 b) 315 c) 512 d) None of these

2. If $m = {}^nC_2$, then ${}^mC_2$ is equal to
a) $3\,{}^nC_4$ b) ${}^{n+1}C_4$ c) $3\,{}^{n+1}C_4$ d) $3.\ {}^{n+1}C_3$

3. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty?
a) 50 b) 100 c) 150 d) 200

4. The interior angles of a regular polygon measure 160° each. The number of diagonals of the polygon are
a) 97 b) 105 c) 135 d) 146

5. There are n number of sets and m number of people have to be seated, then how many ways are possible to do this $(m < n)$?
a) ${}^nP_m$ b) ${}^nC_m$ c) ${}^nC_n \times (m-1)!$ d) ${}^{n-1}P_{m-1}$

6. $\sum_{r=0}^{m} {}^{n+r}C_n$ is equal to
a) ${}^{n+m+1}C_{n+1}$ b) ${}^{n+m+2}C_n$ c) ${}^{n+m+3}C_{n-1}$ d) None of these

7. The set $S = \{1, 2, 3, \dots, 12\}$ is to be partitioned into three sets A, B, C of equal size
Thus, $A \cup B \cup C = S$
$A \cup B = B \cup C = A \cup C = \emptyset$
The number of ways to partition S is
a) $12!/3!\,(4!)^3$ b) $12!/3!\,(3!)^4$ c) $12!/(4!)^3$ d) $12!\,(3!)^4$

8. The number of selecting at least 4 candidates from 8 candidates is
a) 270 b) 70 c) 163 d) None of these

9. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is
a) 360 b) 192 c) 96 d) 48

10. There are 5 roads leading to0 a town from a village. The number of different ways in which a village can go to the town and return back, is
a) 20 b) 25 c) 5 d) 10

11. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible, is
a) 6 b) 7 c) 8 d) 9

12. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then
a) $n = 8, r = 4$ b) $n = 9, r = 3$ c) $n = 7, r = 5$ d) None of these

13. The total number of ways in which 11 identical apples can be distributed among 6 children is
a) 252 b) 462 c) 42 d) None of these

14. A polygon has 44 diagonals, then the number of its sides are
a) 11 b) 7 c) 8 d) None of these

15. The number of ways in which 12 balls can be divided between two friends, one receiving 8 an the other 4, is
a) $\frac{12!}{8!4!}$ b) $\frac{12!2!}{8!4!}$ c) $\frac{12!}{8!4!2!}$ d) None of these
16. Assuming that no two consecutive digits are same. The number of n digit numbers is
a) $n!$ b) 9! c) 9^n d) n^9
17. The figure 4, 5, 6, 7, 8 are written in every possible order. The number of numbers greater than 56000 is
a) 72 b) 96 c) 90 d) 98
18. p points are chosen on each of the three coplanar lines. The maximum number of triangles formed with vertices at these points is
a) $p^3 + 3p^2$ b) $\frac{1}{2}(p^3 + p)$ c) $\frac{p^2}{2}(5p - 3)$ d) $p^2(4p - 3)$
19. Number of divisors of the form $(4n + 2), n \geq 0$ of the integer 240 is
a) 4 b) 8 c) 10 d) 3
20. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
a) $7.\ {}^6C_4.\ {}^8C_4$ b) $8.\ {}^6C_4.\ {}^7C_4$ c) $6.7.\ {}^8C_4$ d) $6.8.\ {}^7C_4$
21. Let l_1 and l_2 be two lines intersecting at P. If A_1, B_1, C_1 are points on l_1 and A_2, B_2, C_2, D_2, E_2 are points on l_2 and if none of these coincides with P, then the number of triangles formed by these eight points, is
a) 56 b) 55 c) 46 d) 45
22. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. Then number of ways in which this can be done, is
a) 3 b) 36 c) 66 d) 108
23. The number of words that can be formed out of the letters of the words 'ARTICE' so that the vowels occupy even places, is
a) 574 b) 36 c) 754 d) 144
24. The value of $2^n[1.3.5 \dots (2n-3)(2n-1)]$ is
a) $\frac{(2n)!}{n!}$ b) $\frac{(2n)!}{2^n}$ c) $\frac{n!}{(2n)!}$ d) None of these
25. The number of ways in which one can post 5 letters in 7 letters boxes is
a) 35 b) ${}^7P_5$ c) 7^5 d) 5^7
26. A car will hold 2 in the front seat and 1 in the rear seat. If among 6 persons 2 can drive, then number of ways in which the car can be filled, is
a) 10 b) 20 c) 30 d) None of these
27. The number of ways of selecting 10 balls from unlimited number of red, black, white and green balls is
a) 286 b) 84 c) 715 d) None of these
28. In a college examination, a candidates is required to answer 6 out of 10 questions which are divided into two sections each containing 5 questions. Further the candidate is not permitted to attempt more than 4 questions from either of the section. The number of ways in which he can make up a choice of 6 questions, is

a) 200 b) 150 c) 100 d) 50

29. In a cricket championship there are 36 matches. The number of terms, if each plays 1 match with other are
a) 9 b) 10 c) 8 d) 12

30. The number of all four digit numbers which are divisible by 4 that can be formed from the digits 1,2,3,4 and 5 is
a) 125 b) 30 c) 95 d) None of these

31. The number of committees of 5 persons consisting of at least one female member, that can be formed from 6 males and 4 females, is
a) 246 b) 252 c) 6 d) None of these

32. The number of ways that 8 beads of different colures be string as a necklace, is
a) 2520 b) 2880 c) 5040 d) 4320

33. 9 balls are to be placed in 9 boxes and 5 of the balls cannot fit into 3 small boxes. The number of ways of arranging one ball in each of the boxes is
a) 18720 b) 18270 c) 17280 d) 12780

34. The expression ${}^nC_r + 4\cdot {}^nC_{r-1} + 6\cdot {}^nC_{r-2} + 4\cdot {}^nC_{r-3} + {}^nC_{r-4}$ equals
a) ${}^{n+4}C_r$ b) $2\cdot {}^{n+4}C_{r-1}$ c) $4\cdot {}^nC_r$ d) $11\cdot {}^nC_r$

35. The total number of natural numbers of six digits that can be made with digits 1, 2, 3, 4, if all digits are to appear in the same number at least once, is
a) 1560 b) 840 c) 1080 d) 480

36. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then the number of ways in which they can be seated, is
a) $\frac{m!\,(m+1)!}{(m-n+1)!}$ b) $\frac{m!\,(m-1)!}{(m-n+1)!}$ c) $\frac{(m-1)!\,(m+1)!}{(m-n+1)!}$ d) None of these

37. If $a, b, c \in N$,The anumber of points having position vector $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ such that $6 \le a + b + c \le 10$, is
a) 110 b) 116 c) 120 d) 127

38. If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, then the value of n is
a) 20 b) 19 c) 18 d) 17

39. There are 10 points in a plane, out of these 6 are collinear. The number of triangles formed by joining these points is
a) 100 b) 120 c) 150 d) None of these

40. A dictionary is printed consisting of 7 letters words only that can be made with a letters of the word CRICKET. If the words are printed at the alphabetical order as in an ordinary dictionary, then the number of words before the word CRICKET is
a) 530 b) 480 c) 531 d) 481

41. A man has 7 friends. In how many ways he can invite one or more of them for a tea party?
a) 128 b) 256 c) 127 d) 130

42. In how many ways can Rs 16 be divided into 4 persons when none of them get less than Rs 3?
a) 70 b) 35 c) 64 d) 192

43. The number of divisors of the number of 38808 (excluding 1 and the number itself)is
a) 70 b) 72 c) 71 d) None of these

44. The number of 4-digit even numbers that can be formed using 0,1,2,3,4,5,6 without repetition is
a) 120 b) 300 c) 420 d) 20

45. If ${}^nP_r = 30240$ and ${}^nC_r = 252$, then the ordered pair (n, r) is equal to
a) $(12, 6)$ b) $(10, 5)$ c) $(9, 4)$ d) $(16, 7)$

46. ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$ is equal to
a) ${}^{n+1}C_r$ b) ${}^{n+1}C_{r+1}$ c) ${}^{n+2}C_r$ d) ${}^{n+2}C_{r+1}$

47. How many nine digit numbers can be formed by using the digits 2,2,3,3,5,5,8,8,8 so that the odd digits occupy even positions?
a) 7560 b) 180 c) 16 d) 60

48. The letters of the word RANDOM are written in all possible orders and these words are written out as in a dictionary, then the rank of the word RANDOM is
a) 614 b) 615 c) 613 d) 616

49. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then the number of such arrangement is
a) At least 500 but less than 750
b) At least 750 but less than 1000
c) At least 1000
d) Less than 500

50. Three dice are rolled. The number of possible outcomes in which at least one die shows 5 is
a) 215 b) 36 c) 125 d) 91

Answer Key

1 **(d)**

We have,

Required number of ways $= (2+1)(3+1)(4+10-1) = 59$

2 **(c)**

Given, $m = {}^nC_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$

Now, ${}^nC_2 = \frac{m!}{2!(m-2)!} = \frac{m(m-1)}{2}$

$= \frac{\frac{n(n-1)}{2}\cdot\left(\frac{n^2-n-2}{2}\right)}{2}$

$= \frac{(n+1)n(n-1)(n-2)}{8}$

$= 3.\ {}^{n+1}C_4$

3 **(c)**

Let the boxes be marked as A, B, C. We have to ensure that no box remains empty and all five balls have to put in. There will be two possibilities.

(i) Any two box containing one ball each and 3rd box containing 3 balls. Number of ways

$= A(1)B(1)C(3)$

$= {}^5C_1.{}^4C_1.{}^3C_3 = 5.4.1 = 20$

Since, the box containing 3 balls could be any of the three aboxes A, B, C. Hence, the required number of ways $20 \times 3 = 60$

(ii) Any two box containing 2 balls each and 3rd containing 1 ball, the number of ways

$= A(2)B(2)C(1) = {}^5C_2.\ {}^3C_2.\ {}^1C_1$

$= 10 \times 3 \times 1 = 30$

Since, the box containing 1 ball could be any of the three boxes A, B, C. Hence, The required number of ways

$= 30 \times 3 = 90$

Hence, total number of ways $= 60 + 90 = 150$

4 **(c)**

Let n be the number of sides of the polygon

$n.160^\circ = (n-2).180^\circ$

$\Rightarrow 20^\circ.n = 360^\circ$

$\therefore\ n = 18$

Then number of diagonals = ${}^{18}C_2 - 18 = 153 - 18 = 135$

5 **(a)**
Required number of ways = ${}^{n}C_m \times m! = {}^{n}P_m$

6 **(a)**
$\sum_{r=0}^{m} {}^{n+r}C_n = \sum_{r=0}^{m} {}^{n+r}C_r$
$= {}^{n}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \ldots + {}^{n+m}C_m$
$= {}^{n+1}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \ldots + {}^{n+m}C_m$
$[\because {}^{n+1}C_0 = {}^{n}C_0]$
$= {}^{n+2}C_1 + {}^{n+2}C_2 + \ldots + {}^{n+m}C_m$
$= {}^{n+m}C_{m-1} + {}^{n+m}C_m$
$= {}^{n+m+1}C_m \qquad [\because {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+r}C_r]$
$= {}^{n+m+1}C_{n+1} \qquad [\because {}^{n}C_r = {}^{n}C_{n-r}]$

7 **(c)**
Required number of ways
$= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4$
$= \frac{12!}{8! \times 4!} \times \frac{8!}{4! \times 4!} \times 1 = \frac{12!}{(4!)^3}$

8 **(c)**
Required number of selections
$= {}^{8}C_4 + {}^{8}C_5 + {}^{8}C_6 + {}^{8}C_7 + {}^{8}C_8$
$= 70 + 56 + 28 + 8 + 1 = 163$

9 **(c)**
Arrange the letter of the word COCHIN as in the order of dictionary CCHINO
Which number of words with the two C's occupying first and second place= 4!
Number of words starting with CH, CI, CN is 4! each
$\therefore$ Total number of ways = 4! + 4! + 4! + 4! = 96
There are 96 words before COCHIN

10 **(b)**
The villagers can go to the town in ${}^{5}C_1$ ways and they return back in ${}^{5}C_1$ ways.
$\therefore$ Total number of ways= ${}^{5}C_1 \times {}^{5}C_1 = 25$

11 **(b)**
The number of distinct n-digit numbers to be formed using digits 2, 5 and 7 is 3^n. We have to find n so that

$3^n \geq 900 \Rightarrow 3^{n-2} \geq 100$

$\Rightarrow n-2 \geq 5 \Rightarrow n \geq 7$

So the least value of n is 7

12 **(b)**

We have,

${}^nC_{r-1} = 36,\ {}^nC_r = 84,\ {}^nC_{r+1} = 126$

$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{84}{36}$ and $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{126}{84}$

$\Rightarrow \frac{n-r+1}{r} = \frac{7}{3}$ and $\frac{n-r}{r+1} = \frac{3}{2}$

$\Rightarrow 3n - 10r + 3 = 0$ and $2n - 5r - 3 = 0 \Rightarrow r = 3, n = 9$

13 **(d)**

The required number is the coefficient of x^{11} in $(1 + x + x^2 + \cdots + x^{11})^6 = {}^{11+6-1}C_{6-1} = {}^{16}C_5$

14 **(a)**

Let number of sides of polygon=n

$\Rightarrow \quad {}^nC_2 - n = 44 \qquad$ [given]

$\Rightarrow \quad \frac{n!}{2!\,(n-2)!} - n = 44$

$\Rightarrow \quad n(n-1) - 2n = 88$

$\Rightarrow \quad n^2 - 3n - 88 = 0$

$\Rightarrow \quad (n-11)(n+8) = 0 \quad \Rightarrow \quad n = 11, -8$

Since, sides cannot be negative

$\therefore \quad n = 11$

15 **(b)**

12 balls can be distributed between two friends A and B in two ways

(i) Friend A receives 8 and B receives 4

(ii) Friend B receives 8 and A receives 4

$\therefore$ Required number of ways $= \frac{12!}{8!\,4!} + \frac{12!}{4!\,8!} = 2\left(\frac{12!}{8!\,4!}\right)$

16 **(c)**

Digit at the extreme left can be chosen by 9 ways as zero cannot be the first digit. Now for the second digit it can be done in 9 ways as consecutive digits are not same. And this is same for next digits. Hence, number of ways are

$9 \times 9 \times 9 \times \ldots\ldots \times n$ times$= 9^n$

17 **(c)**

The number forms by the figure 4, 5, 6, 7, 8 which is greater than 56000 is in two cases.

Case I Let the ten thousand digit place number be greater than 5. The number of numbers

$= 3 \times 4 \times 3 \times 2 \times 1 = 72$

Case II Let the ten thousand digit number be 5 and thousand digit number be either 6 or greater than 6. Then, the number of numbers $= 3 \times 3 \times 2 \times 1 = 18$

$\therefore$ Required number of ways $= 72 + 18 = 90$

18 **(d)**

Total number of points in a plane is $3p$

$\therefore$ Maximum number of triangles

$= {}^{3p}C_3 - 3.\, {}^{p}C_3$

[here, we subtract those triangles which points are in a line]

$$= \frac{(3p)!}{(3p-3)!\,3!} - 3.\frac{p!}{(p-3)3!}$$

$$= \frac{3p(3p-1)(3p-2)}{3 \times 2} - \frac{3 \times p(p-1)(p-2)}{3 \times 2}$$

$$= \frac{p}{2}[9p^2 - 9p + 2 - (p^2 - 3p + 2)] = p^2(4p-3)$$

19 **(a)**

$\because \quad 240 = 2^4.3.5$

$\therefore$ Total number of divisors=(4+1)(2)(2)=20

Out of these 2, 6, 10 and 30 are of the from $4n + 2$

20 **(a)**

Given word is MISSISSIPPI

Here, I=4 times, S=4 times, P=2 times, M=1 time

_M_I_I_I_I_P_P_

Required number of words $= {}^{8}C_4 \times \frac{7!}{4!2!}$

$$= {}^{8}C_4 \times \frac{7 \times 6!}{4!\,2!} = 7.\,{}^{8}C_4.\,{}^{6}C_4$$

21 **(d)**

If triangle is formed including point 'P' the other points must be one from l_1 and other point from l_2. Number of triangle formed with $P = {}^{3}C_1 \times {}^{5}C_1 = 15$ ways

When P is not included.

Number of triangle formed

$= {}^{3}C_2 \times {}^{5}C_1 + {}^{3}C_1 \times {}^{5}C_2 = 15 + 15 = 30$

Total number of triangles=15+30=45

22 **(d)**

The number of ways in which two balls form urn A and two balls from urn B can be selected

$= {}^{3}C_2 \times {}^{9}C_2 = 3 \times 36 = 108$

23 **(d)**

The word 'ARTICLE' has 3 vowels and 4 consonants and according to problem we have to put the 3 vowels on 3 even places and 4 consonants in the remaining places.

$\therefore$ The required number of ways

$= 3! \times 4! = 6 \times 24 = 144$

24 **(a)**

$[1.3.5 \dots\dots (2n-1)]2^n$

$$= \frac{1.2.3.4.5.6 \dots\dots (2n-1)(2n)2^n}{2.4.6 \dots\dots 2n}$$

$$= \frac{(2n)!\,2^n}{2^n(1.2.3 \dots\dots n)} = \frac{(2n)!}{n!}$$

25 **(c)**

Each letter can be posted in any one of the 7 letter boxes. So, required number of ways =
$7 \times 7 \times 7 \times 7 \times 7 = 7^5$

26 **(b)**

Since, 2 persons can drive the car, therefore we have to select 1 from these two. This can be done in ${}^2C_1$ways. Now, from the remaining 5 persons we have to select 2 which can be done in ${}^5C_2$ ways.

Therefore, the required number of ways in which the car can be filled

$= {}^5C_2 \times {}^2C_1 = 10 \times 2 = 20$

27 **(a)**

We have,
Required number of ways
$=$ Coefficient of x^{10} in $(1 + x + x^2 + \cdots)^4$
$=$ Coefficient of x^{10} in $(1-x)^{-4}$
$= {}^{10+4-1}C_{4-1} = {}^{13}C_3 = 286$

28 **(a)**

The required number of ways
$= {}^5C_4 \cdot {}^5C_2 + {}^5C_3 + {}^5C_2 \cdot {}^5C_4$
$= 50 + 100 + 50 = 200$

29 **(a)**

Let n be the number of terms
$\because \quad {}^nC_2 = 36$
$\Rightarrow \quad \frac{n(n-1)}{1.2} = 36$
$\Rightarrow \quad n(n-1) = 72 = 9 \times 8$
$\Rightarrow \quad n = 9$

30 **(a)**

The number formed will be divisible by 4 if the number formed by the two digits on the extreme right is divisible by 4 i.e. it should be 12,24,32,52,44
The number of numbers ending in $12 = 5 \times 5$
The number of numbers ending in $24 = 5 \times 5$

The number of numbers ending in 32 $= 5 \times 5$
The number of numbers ending in 52 $= 5 \times 5$
The number of numbers ending in 44 $= 5 \times 5$
Thus, the required number
$= 5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 5 = 125$

32 **(a)**
8 different beads can be arranged in circular form in $(8-1)! = 7!$ ways. Since, there is no distinction between the clockwise and anticlockwise arrangement. So, the required number of arrangements$= \frac{7!}{2} = 2520$

33 **(c)**
Required number of arrangements
$= {}^6P_5 \times 4! = 720 \times 24 = 17280$

34 **(a)**
We have,
$${}^nC_r + 4 \cdot {}^nC_{r-1} + 6 \cdot {}^nC_{r-2} + 4 \cdot {}^nC_{r-3} + {}^nC_{r-4}$$
$$= ({}^nC_r + {}^nC_{r-1}) + 3({}^nC_{r-1} + {}^nC_{r-2}) + 3({}^nC_{r-2} + {}^nC_{r-3}) + ({}^nC_{r-3} + {}^nC_{r-4})$$
$$= {}^{n+1}C_r + 3 \cdot {}^{n+1}C_{r-1} + 3 \cdot {}^{n+1}C_{r-2} + {}^{n+1}C_{r-3}$$
$$= ({}^{n+1}C_r + {}^{n+1}C_{r-1}) + 2({}^{n+1}C_{r-1} + {}^{n+1}C_{r-2}) + ({}^{n+1}C_{r-2} + {}^{n+1}C_{r-3})$$
$$= {}^{n+2}C_r + 2 \cdot {}^{n+2}C_{r-1} + {}^{n+2}C_{r-2}$$
$$= ({}^{n+2}C_r + {}^{n+2}C_{r-1}) + ({}^{n+2}C_{r-1} + {}^{n+2}C_{r-2})$$
$$= {}^{n+3}C_r + {}^{n+3}C_{r-1} = {}^{n+4}C_r$$

35 **(a)**
There can be two types of numbers
(i) any one of the digits 1,2,3,4 repeats thrice and the remaining digits only once i.e. of the type 1,2,3,4,4,4
(ii) any two of the digits 1,2,3,4 repeat twice and the remaining two only once i.e. of the type 1,2,3,4,4
Number of numbers of the type 1 2 3 4 4 4
$$= \frac{6\,!}{3\,!} \times {}^4C_1 = 480$$
Number of numbers of the type 1 2 3 3 4 4
$$= \frac{6\,!}{2!\ 2!} \times {}^4C_2 = 1080$$
So, the required number $= 480 + 1080 = 1560$

36 **(a)**
First arrange m men in a row in $m!$ ways. Since, $n < m$ and no two women can sit together in any one of the $m!$ arrangement, there are $(m+1)$ places in which n women can be arranged in ${}^{m+1}P_n$ ways.

$\therefore$ The required number of arrangements of m men and n women $(n < m)$

$= m!^{m+1}P_n = \frac{m!\,(m+1)!}{(m-n+1)!}$

37 **(a)**

Given, $6 \leq a+b+c \leq 10$

$\therefore\ a+b+c = 6, 7, 8, 9, 10$

Here $a \geq 1, b \geq 1, c \geq 1$

$\therefore$ Required number of ways

$= {}^5C_2 + {}^6C_2 + {}^7C_2 + {}^8C_2 + {}^9C_2$

$= 110$

38 **(b)**

We have,

${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$

$\Rightarrow \frac{(n+2)\,!\,(n-6)\,!}{(n-6)\,!\,(n-2)\,!\,8\,!} = \frac{57}{16}$

$\Rightarrow (n+2)(n+1)n(n-1) = 143640$

$\Rightarrow (n^2+n-2)(n^2+n) = 143640$

$\Rightarrow (n^2+n)^2 - 2(n^2+n) + 1 = 143641$

$\Rightarrow (n^2+n-1)^2 = (379)^2$

$\Rightarrow n^2+n-1 = 379 \qquad [\because n^2+n-1 > 0]$

$\Rightarrow n^2+n-380 = 0$

$\Rightarrow (n+20)(n-19) = 0 \Rightarrow n = 19 \quad [\because n \text{ is not negative}]$

39 **(a)**

A triangle is obtained by joining three non-collinear points. So number of triangles on joining 3 points out of 10 points $= {}^{10}C_3$. But, 6 points are collinear and on joining any three out of these 6, we do not obtain a triangle

Hence, the required number of triangles

$= {}^{10}C_3 - {}^6C_3 = 120 - 20 = 100$

40 **(a)**

$\because$ Given word is CRICKET

total number of letters are 7 out of which two letters 'C' are count as one

$\therefore$ Required number of ways of words before the word CRICKET$= 5! \times 4 + 2 \times 4! + 2!$

$= 480 + 48 + 2 = 530$

41 **(c)**

A man has two options for every friend either they invited it or not.

$\therefore$ Required number of ways $= 2^7 - 1 = 127$

[Since, we have to subtract those cases in which he does not invite any friend *ie*, ${}^nC_0 = 1$]

Alternate Solution

Required number of ways $= {}^7C_1 + {}^7C_2 + {}^7C_3 + \cdots + {}^7C_7$

$= 2^7 - 1$

42 **(b)**

Required number of ways

$=$coefficient of x^{16}in $(x^3 + x^4 + x^5 + \ldots\ldots + x^{16})^4$

$=$coefficient of x^{16}in $x^{12}(1 + x + x^2 + \ldots\ldots\ldots + x^{12})^4$

$=$coefficient of x^4 in $(1 - x^{13})^4(1 - x)^{-4}$

$=$coefficient of x^4 in $(1 - 13x^5 + \ldots)$

$$\times \left[1 + 4x + \ldots\ldots + \frac{(r+1)(r+2)(r+3)}{3!}x^r\right]$$

$$= \frac{(4+1)(4+2)(4+3)}{3!} = 35$$

43 **(a)**

Since, $38808 = 2^3 \times 3^2 \times 7^2 \times 11^1$

$\therefore$ Number of divisors $= 4 \times 3 \times 3 \times 2 - 2$

$= 72 - 2 = 70$

44 **(c)**

An even number has an even digit at unit place

$\therefore$ Required number of even numbers

$=$ Number of even numbers having 0 at unit's place

$+$ Number of even numbers having a non-zero digit at unit's place

$= {}^6C_3 \times 3\,! \times 1 + {}^3C_1({}^6C_3 \times 3\,! - {}^5C_2 \times 2\,!)$

$= 120 + 3 \times (120 - 20) = 420$

45 **(b)**

Given, ${}^nC_r = 30240$ and ${}^nC_r = 252$

$\frac{n!}{(n-r)!} = 30240$ and $\frac{n!}{(n-r)!r!} = 252$

$$\Rightarrow \quad r! = \frac{30240}{252} = 120 \quad \Rightarrow \quad r = 5$$

$$\therefore \quad \frac{n!}{(n-5)!} = 30240$$

$\Rightarrow \quad n(n-1)(n-2)(n-3)(n-4)$
$= 10(10-1)(10-2)(10-3)(10-4)$
$\Rightarrow \quad n = 10$
Hence, required ordered pair is (10, 5)

46 **(c)**

$^nC_r + 2\,^nC_{r-1} + \,^nC_{r-2} = \,^nC_r + \,^nC_{r-1} + \,^nC_{r-1} + \,^nC_{r-2}$

$= ^{n+1}C_r + \,^{n+1}C_{r-1} = \,^{n+2}C_r$

47 **(d)**

4 odd digits 3,3,5,5 can occupy 4 even places in $\frac{4!}{2!2!}$ ways and 5 even digits 2,2,8,8,8 can occupy 5 odd places in $\frac{5!}{3!2!}$ ways

$\therefore$ Required number of nine digit numbers

$= \frac{4!}{2!2!} \times \frac{5!}{3!2!} = 60$

48 **(a)**

In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with A, D, M, N, O, R in order. A will occur in the first place as often as there are ways of arranging the remaining 5 letters all at a time. i.e. A will occur 5! Times. D, M, N, O will occur in the first place the same number of times. So,

Number of words starting with $A = 5! = 120$
Number of words starting with $D = 5! = 120$
Number of words starting with $M = 5! = 120$
Number of words starting with $N = 5! = 120$
Number of words starting with $O = 5! = 120$
Number of words beginning with RAD or RAM, is 3!
Now the words beginning with $'RAN'$ must follow
First one is $RANDMO$ and the next one is $RANDOM$
$\therefore$ Rank of $RANDOM = (5!)5 + (3!)2 + 2 = 614$

49 **(c)**

The number of ways in which 4 novels can be selected
$= \,^6C_4 = 15$
The number of ways in which 1 dictionary can be selected
$= \,^3C_1 = 3$
4 novels can be arranged in 4! ways
$\therefore$ The total number of ways
$= 15 \times 4! \times 3 = 15 \times 24 \times 3 = 1080$

50 **(d)**

Required number of possible outcomes
= Total number of possible outcomes – Number of possible outcomes in which 5 does not appear on any dice
$= 6^3 - 5^3 = 216 - 125 = 91$

Answer Key

Ques.	1	2	3	4	5	6	7	8	9	10
Ans.	D	C	C	C	A	A	C	C	C	B
Ques.	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	D	A	B	C	C	D	A	A
Ques.	21	22	23	24	25	26	27	28	29	30
Ans.	D	D	D	A	C	B	A	A	A	A
Ques.	31	32	33	34	35	36	37	38	39	40
Ans.	A	A	C	A	A	A	A	B	A	A
Ques.	41	42	43	44	45	46	47	48	49	50
Ans.	C	B	A	C	B	C	D	A	C	D